Temperature of a granular material ''fluidized'' by external vibrations

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(Received 1 December 1997)

The scaling for the temperature of a granular material ''fluidized'' by external vibrations is determined in the limit where the dissipation of energy in a collision due to inelasticity, or between successive collisions due to viscous drag, is small compared to the energy of the particles. An asymptotic scheme is used, where the dissipation of energy is neglected in the leading approximation, and the Boltzmann equation for the system is identical to that for a gas at equilibrium in a gravitational field. The density variation in the ''fluidized'' material is given by the Boltzmann distribution, and the velocity distribution is given by the Maxwell-Boltzmann distribution. However, the ''temperature'' of the material is not specified by thermodynamic considerations, but is determined by a balance between the source of energy due to the vibrating surface and the dissipation of energy. This balance indicates that the dependence of temperature on the amplitude of the vibrating surface is sensitively dependent on the mechanism of dissipation (inelastic collisions or viscous drag), and also on whether the amplitude function for the velocity of the vibrating surface is symmetric or asymmetric about zero velocity. However, the temperature turns out to have the same functional dependence on the properties of the system in two and three dimensions. [S1063-651X(98)08004-0]

PACS number(s): 81.05.Rm, 46.10.+z, 05.70.Ln, 62.90.+k

I. INTRODUCTION

There has been increasing interest in the dynamics of granular materials which are set into motion by external vibrations. A typical configuration involves a (two-dimensional or three-dimensional) bed of particles supported on a horizontal plate which is vibrated with a certain amplitude function and frequency. Under certain conditions of vibration amplitude and frequency, the material becomes ''fluidized,'' and the motion of the individual particles is qualitatively similar to the motion of molecules in a gas at equilibrium. However, there is a significant difference between a fluidized granular material and a gas at equilibrium — the temperature of the gas, which gives the magnitude of the velocity fluctuations, is a thermodynamic property specified by ambient conditions, while the temperature of a granular material depends on the balance between the source of energy, due to the vibrating surface, and the dissipation of energy due to inelastic collisions or viscous drag on the particles.

Some unusual features in granular materials subject to vibrations, such as the formation of convection cells [1] and the presence of compression and transverse waves [2] have been reported. The transition from a liquidlike condensed state to a gaslike fluidized state has also been studied [2–4]. More recently, there have been simulations that have obtained the properties of the fluidized state, such as the granular temperature and the variation in the height of the center of mass [5,6]. Experimental information on the velocity distribution functions has been obtained using high speed photography [7,8] for a two-dimensional array of spheres. These experiments have yielded information on the moments of the velocity distribution and the spatial pair correlation function in these materials. Though the velocity distribution function is similar in shape to the Maxwell-Boltzmann distribution for a gas at equilibrium, there have been significant systematic differences observed in the experiments. A significant feature of the experimental and simulation results carried out so far is that there is no unanimity regarding the dependence of the granular temperature on the amplitude of vibration of the surface and the inelasticity of the particles. The simulations of Luding, Hermann, and Blumen [5] indicate that the granular temperature varies as $T \propto U_0^{1.5}$ while the experimental results of Warr *et al.* [8] show a dependence of the form $T \propto U_0^{1.41}$ where U_0 is the amplitude of the velocity of the vibrating plate. However, this type of scaling has not been obtained by theoretical analyses, and theoretical models for the one-dimensional vibration of a particle show a scaling relation of the form $T \propto U_0^2$. The scaling relations are derived in the present analysis in the limit where the dissipation of energy of a particle in a collision or between successive collisions is small compared to the energy of the particles. The results indicate that the scaling of the temperature is sensitive to the mechanism of energy dissipation (inelastic collisions or viscous drag) as well as to the form of the amplitude function of the vibrating surface.

In this analysis, scaling relations are derived in the limit where the maximum velocity of the vibrating plate is small compared to the velocity fluctuations in the granular medium. It is shown a little later that in this limit, the dissipation of energy during a collision due to inelasticity, or between successive collisions due to viscous drag, is small compared to the energy of a particle. The parameter regime where the above approximation is valid is determined self-consistently a little later.

Two density regimes are considered, depending on the magnitude of the parameter Nr, where N is the number of particles per unit width of the vibrated material and r is the radius of a particle. (Note that $2Nr$ is the number of monolayers of particles when the granular material is at rest.) In the ''dense limit'' $Nr \geqslant 1$, a particle collides with many other particles in between successive collisions with the vibrating surface. In this case, the distribution function is obtained

using an asymptotic analysis. In the leading order approximation, the system is considered nondissipative, and the distribution function for the velocity of the particles is identical to the Maxwell-Boltzmann distribution for a gas of hard spheres in a gravitational field. However, the ''temperature'' of the material is not fixed from thermodynamic considerations, but has to be determined by a balance between the dissipation of energy due to inelastic collisions or viscous drag and the source due to the particle collisions with the vibrating surface. The source and dissipation of energy are calculated using methods from the kinetic theory of gases, and the velocity fluctuations at steady state are estimated by balancing the source and dissipation of energy.

In the ''dilute limit'' $Nr \ll 1$ (which corresponds to a ''Knudsen flow''), the frequency of particle collisions with the vibrating surface is large compared to the frequency of binary collisions. In the leading approximation, binary collisions are neglected and the velocity distribution for a particle colliding with a vibrating surface is derived. For a symmetric vibrator, the results show that the velocity distribution is identical to one-dimensional Maxwell-Boltzmann distribution for the case where dissipation is due to inelastic collisions, but could be very different when the dissipation is due to viscous drag. For an asymmetric vibrator, the single particle distribution is a δ function at a specific velocity for inelastic collisions and for viscous drag.

II. DENSE LIMIT

In the leading approximation, the system is considered nondissipative, and the distribution function for the particle velocities is identical to that for a gas at equilibrium in a gravitational field. The leading order density and velocity distributions are Boltzmann distributions:

$$\rho_0 = \frac{Nmg}{T} \exp\left(- \frac{mgz}{T} \right), \tag{1}$$

$$f_0(\mathbf{u}) = \left(\frac{m}{2\pi T} \right)^{(d/2)} \exp\left(\frac{-m\mathbf{u}^2}{2T} \right), \tag{2}$$

where d is the dimension, N is the number of particles per unit length in the horizontal direction for a two-dimensional system, and the number of particles per unit area in the horizontal plane for a three-dimensional system, g is the acceleration due to gravity, z is the vertical distance in the direction opposite to gravity, T is the temperature in the leading approximation, and m is the mass of the particles. However, the temperature T cannot be determined from the leading order solutions, and has to be determined by equating the rate of increase of energy due to particle collisions with the vibrating surface, and the rate of dissipation of energy due to inelastic collisions or viscous drag.

The rate of dissipation of energy due to inelastic collisions is obtained using methods similar to those used in the kinetic theory of gases. The rate of dissipation of energy, D_I (per unit length in the horizontal direction for a two-dimensional system, and per unit area in the horizontal plane for a three-dimensional system), is determined by considering the dissipation of energy in a collision between two particles with vertical positions and velocities $(z,\mathbf{u})$ and $(z^*,\mathbf{u}^*)$, and doing an ensemble average over the velocities and orientations of the particles.

$$D_I = - \int_0^\infty dz \rho_0(z) \rho_0(z^*) \int d\mathbf{u} \int d\mathbf{u}^* \times \int d\mathbf{k} f_0(z,\mathbf{u}) f_0(z^*,\mathbf{u}^*) (\Delta E) [\sigma(\mathbf{u}-\mathbf{u}^*)\cdot\mathbf{k}], \tag{3}$$

where $\mathbf{k}$ is the orientation of the line joining the centers of the particles with respect to the vertical and is directed from the particle with velocity $\mathbf{u}$ to the particle with velocity $\mathbf{u}^*$, ΔE is the change in the kinetic energy during the collision, and σ is $2r$ in two dimensions and $(2r)^2$ in three dimensions, and r is the radius of a particle. It is also important to note that the integral in Eq. (3) is carried out only for those values of $\mathbf{k}$ for which $\mathbf{k}\cdot(\mathbf{u}-\mathbf{u}^*)$ is positive, since the particles do not collide if $\mathbf{k}\cdot(\mathbf{u}-\mathbf{u}^*)$ is negative. The rate of dissipation of energy is determined using the standard techniques of kinetic theory of gases [9], which have also been used for granular materials in shear flow [10,11]. The rate of dissipation of energy for a system where the distribution function is given by 1 and 2 is

$$D_I = \begin{cases} \sqrt{\pi} r N^2 g (mT)^{1/2} (1-e^2) & \text{in 2 dimensions} \\ 4\sqrt{\pi} r^2 N^2 g (mT)^{1/2} (1-e^2) & \text{in 3 dimensions.} \end{cases} \tag{4}$$

The rate of dissipation of energy due to viscous drag is determined using a drag law of the form

$$\mathbf{a} = -\mu \mathbf{u}, \tag{5}$$

where μ, the drag coefficient, is given by $(6\pi\eta r/m)$ for a system where the drag force is given by Stokes law and the fluid has a viscosity η. The rate of dissipation of energy due to viscous drag is given by

$$D_D = \int_0^\infty dz \rho(z) \int d\mathbf{u} f(z,\mathbf{u}) \mu (\mathbf{u}\cdot\mathbf{u}). \tag{6}$$

The above integral can also be evaluated using the standard techniques of kinetic theory of gases, and the rate of dissipation of energy per unit length in two dimensions and per unit area in three dimensions is given by

$$D_D = \begin{cases} 2\mu NT & \text{in 2 dimensions} \\ 3\mu NT & \text{in 3 dimensions.} \end{cases} \tag{7}$$

The source of energy required to sustain the velocity fluctuations is provided by the particle collisions with the vibrating surface. The flux of energy is calculated by averaging over the distribution function of particle velocities as well as the probability distribution function of the velocities of the vibrating surface. The probability function $P(U)$ is defined such that $P(U)dU$ is the probability of finding the surface with a velocity in the interval dU about U, and

$$\int_{U_{\min}}^{U_{\max}} dU \;\; P(U) = 1. \tag{8}$$

The probability function can be easily determined from the time series of the amplitude of the vibrator. Assuming that

the collisions of the particles with the vibrator are elastic, the initial $\mathbf{u}$ and the final velocity $\mathbf{u}'$ of the particle are given by

$$u_z' - U = -(u_z - U), \quad u_x' = u_x, \quad u_y' = u_y, \tag{9}$$

where x and y are orthogonal coordinates in the horizontal plane. The source of energy due to particle collisions with the vibrating surface is given by

$$S = \rho(0) \int_{U_{\min}}^{U_{\max}} dU \; P(U) \int_{-\infty}^{U} du_z \int_{-\infty}^{\infty} du_x \times \int_{-\infty}^{\infty} du_y (U - u_z) f(0,\mathbf{u}) \Delta E, \tag{10}$$

where ΔE is the change in energy of the particle, and the factor $\rho(0)(U-u_z)f(0,\mathbf{u})$ gives the normal flux of particles at the surface. The leading contribution to the energy source is obtained by inserting $f(0,\mathbf{u}) = f_0(\mathbf{u})$ and $\rho(0) = \rho_0(0)$ in Eq. (10). This integral is difficult to evaluate without knowing the form of the distribution function $P(U)$, but the integral can be evaluated using an expansion in the parameter (mU^2/T) in the limit $mU^2 \ll T$. It is shown a little later that this corresponds to the limit where the energy dissipation in a collision due to inelasticity, or between successive collisions due to viscous drag, is small compared to the energy of a particle. In this limit, the leading order terms in the equation for the source of energy are

$$S_0 = \frac{Nmg}{T}\left(T\langle U \rangle + 2\sqrt{\frac{2}{\pi}}\sqrt{mT}\langle U^2 \rangle + O(mU^3) \right). \tag{11}$$

In the present analysis, we distinguish between two types of vibrations. For symmetric vibrations, the probability distribution for the velocity of the surface $P_s(U)$ is symmetric about $U=0$, and the average velocity of the surface $\langle U \rangle = 0$. A sinusoidal amplitude variation $A = A_0 \cos(\omega t)$ is an example of a symmetric amplitude function with maximum velocity U_0. For symmetric amplitude functions $\langle U \rangle = 0$, and the leading order term in Eq. (11) is proportional to $\langle U^2 \rangle$. For asymmetric vibrations for which the probability distribution $P_a(U)$ is asymmetric about $U=0$, and $\langle U \rangle \neq 0$. A sawtooth amplitude function $A = A_0[(t/t_0) - \mathrm{int}(t/t_0)] - A_0/2$ is an asymmetric amplitude function, where $\mathrm{int}(x)$ denotes the highest integer that is lower than x. In the latter case, the velocity has a constant positive value A_0/t_0, except for integer values of t/t_0, where the velocity is undefined since the amplitude is discontinuous. During the time that the velocity is undefined, there are no collisions between the particle and the surface, and collisions occur only when the surface is moving with a constant velocity A_0/t_0. Consequently, for asymmetric amplitude functions, $\langle U \rangle \neq 0$, and the leading order term in Eq. (11) is proportional to $\langle U \rangle$.

A balance between S_0, Eq. (11), and the rate of dissipation of energy (4) or (7) gives the relations for the temperatures shown in Table I. Before examining these temperature relations, it is useful to recall that this analysis is restricted to the limit where the square of the amplitude of the velocity of the surface is small compared to the temperature, and to examine the parameter regimes where this assumption is valid. For systems where dissipation is due to inelastic collisions, it is apparent that this condition is valid for $1-e^2 \ll 1$ (for $Nr \sim 1$ in two dimensions or $Nr^2 \sim 1$ in three dimensions), which corresponds to the limit where the coefficient of restitution is close to 1 and the dissipation of energy in a collision due to inelasticity is small compared to the energy of the particle. For systems where the dissipation is due to viscous drag, it can be inferred that this condition is valid in the limit where $(\mu T^{1/2}/m^{1/2}g) \ll 1$. Since the fluctuating velocity of the particles scales as $(T/m)^{1/2}$, the rate of dissipation of energy scales as $\mu T/m$ and the distance between collisions scales as $(r\rho_0)^{-1}$ in two dimensions and $(r^2\rho_0)^{-1}$ in three dimensions, the ratio of the dissipation of energy between successive collisions and the energy of a particle scales as $\mu T^{1/2}/(m^{1/2}Nrg)$ in two dimensions and $\mu T^{1/2}/(m^{1/2}Nr^2g)$ in three dimensions. Therefore the condition that the square of the amplitude of velocity fluctuations is small compared to the temperature corresponds to the limit where the dissipation of energy between successive collisions due to viscous drag is small compared to the energy of a particle for situations where $Nr \sim 1$ in two dimensions and $(Nr^2) \sim 1$ in three dimensions.

TABLE I. Temperature scaling in the dense limit.

Dimension	Dissipation	Vibrator	T
2	Inelastic	Symmetric	$\frac{2\sqrt{2}}{\pi}\frac{m\langle U^2\rangle}{(Nr)(1-e^2)}$
2	Inelastic	Asymmetric	$\frac{m\langle U\rangle^2}{\pi[(Nr)(1-e^2)]^2}$
3	Inelastic	Symmetric	$\frac{\sqrt{2}}{\pi}\frac{m\langle U^2\rangle}{2(Nr^2)(1-e^2)}$
3	Inelastic	Asymmetric	$\frac{m\langle U\rangle^2}{\pi[4(Nr^2)(1-e^2)]^2}$
2	Viscous	Symmetric	$\left[\sqrt{\frac{2}{\pi}}\frac{m^{3/2}\langle U\rangle^2 g}{2\mu}\right]^{2/3}$
2	Viscous	Asymmetric	$\frac{mg\langle U\rangle}{2\mu}$
3	Viscous	Symmetric	$\left[\sqrt{\frac{2}{\pi}}\frac{m^{3/2}\langle U\rangle^2 g}{3\mu}\right]^{2/3}$
3	Viscous	Asymmetric	$\frac{mg\langle U\rangle}{3\mu}$

III. DILUTE LIMIT

In the dilute (Knudsen) limit, the frequency of particle collisions with the vibrating surface is large compared to the frequency of binary collisions. In the leading approximation, the distribution function for a single particle on a vibrating surface is considered. The leading order distribution functions for the cases where dissipation is due to inelastic collisions and viscous drag are considered separately, and the correction to the distribution function due to binary collisions between particles is then estimated.

A. Dissipation due to inelastic collisions

In the collision model for a single particle on a vibrating surface the velocity after collision u_z is related to the velocity before collision u_z' by

$$u_z - U = -e(-u_z' - U), \tag{12}$$

where e is the coefficient of restitution, and U is the velocity of the vibrating surface. Note the requirement $U > -u_z'$ for a particle to collide with the wall. Inserting $\epsilon_I = (1-e)$ in Eq. (12), and retaining terms up to $O(\epsilon_I)$, the equation for u_z' is

$$u_z' - u_z = \epsilon_I u_z + (2-\epsilon_I)U. \tag{13}$$

For an asymmetric vibrator with an amplitude function of the form $A = A_0[(t/t_0) - \mathrm{int}(t/t_0)] - A_0/2$, it can easily be seen that the velocity of the particle at the vibrating surface at steady state is $(1+e)U/(1-e)$, and the distribution function is a δ function at this velocity. The distribution function at any height z can be inferred from this,

$$f(u_z, z) = n\delta\left(u_z - gz - \frac{(1+e)U}{1-e}\right). \tag{14}$$

For a symmetric vibrator, the distribution function can be determined using a flux balance condition. The zero net flux condition across the vibrating surface indicates that the total flux of particles with velocity in the interval du_z' about $-u_z'$ incident on the surface is equal to the flux of particles with velocity in the interval du_z about u_z reflected from the surface. The flux of particles incident on the surface is

$$N_i(u_z)du_z = (U + u_z')F(u_z')du_z'. \tag{15}$$

In the above equation, we have used the symmetry condition $F(-u_z') = F(u_z')$ for the free flight of a particle in the absence of drag forces. Using Eq. (13) to express u_z' in terms of u_z, the flux of particles entering the interval du_z is

$$N_i(u_z) = (1/e)(U + u_z')F(u_z'). \tag{16}$$

The flux of particles reflected from the surface having velocity in the interval du_z about u_z is given by

$$N_r(u_z) = (u_z - U)F(u_z). \tag{17}$$

At steady state, the distribution function is determined from the equation $N_i(u_z) = N_r(u_z)$. This equation relates the distribution function at the velocity before collision with that at the velocity after collision, and is a difference equation for the velocity distribution. This equation is difficult to solve in general, but a solution can be obtained in the limit $\epsilon_I \ll 1$, where the difference between the velocities before and after collision is small compared to the velocity of a particle. In this limit, the distribution function $F(u_z')$ can be expanded in a Taylor series about $u_z' = u_z$:

$$F(u_z') = F(u_z) + (u_z' - u_z)\frac{dF}{du_z} + \frac{(u_z' - u_z)^2}{2}\frac{d^2F}{du_z^2} + O(u_z' - u_z)^3. \tag{18}$$

The above expansion is inserted into the flux balance condition $N_i(u_z) = N_r(u_z)$, and the resulting equation is expanded in the parameter ϵ_I. It is also useful to note at this point that the velocity of the vibrating surface $U \sim \epsilon_I^{1/2} u_z$, and so only terms proportional to U and U^2 in the flux balance condition are retained. The resulting equation is

$$\epsilon_I\left(2Fu_z + u_z^2\frac{dF}{du_z}\right) - 2Uu_z\frac{dF}{du_z} + 2U^2\left(u_z\frac{d^2F}{du_z^2} + \frac{dF}{du_z}\right) = 0. \tag{19}$$

An ensemble average over the distribution of velocities U of the vibrating surface is carried out, to give

$$\epsilon_I\left(2Fu_z + u_z^2\frac{dF}{du_z}\right) + 2\langle U^2\rangle\left(u_z\frac{d^2F}{du_z^2} + \frac{dF}{du_z}\right) = 0. \tag{20}$$

It is useful to express the above equation in terms of a scaled velocity $u_z^* = u_z/(2\langle U^2\rangle/\epsilon_I)^{1/2}$ to obtain the final equation for the velocity distribution function:

$$u_z^* F'' + (u_z^{*2} + 1)F' + 2u_z^* F = 0, \tag{21}$$

where primes denote differentiation with respect to u_z^*. It can also be easily verified that the third and higher order terms in the Taylor series expansion (20) make subdominant contributions to the differential equation for the velocity distribution function, and so the differential equation (21) is correct to leading order in small ϵ_I. Equation (21) can be easily solved to obtain the Maxwell-Boltzmann distribution for F,

$$F = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u_z^{*2}}{2}\right). \tag{22}$$

B. Dissipation due to viscous drag

In this section, the single particle distribution function is derived for the case where particle collisions with the vibrating surface are elastic, but there is dissipation of energy due to viscous drag, and the drag force is linear in the particle velocity as before. The acceleration of the particle is

$$\frac{du_z}{dt} = -g - \mu u_z. \tag{23}$$

If the particle has a velocity u_z after a collision with the vibrating surface, the vertical velocity before the next successive collision in the limit of small dissipation $(\mu u_z/g) \ll 1$ is

$$u_z'' = -u_z + \frac{2\mu u_z^2}{3g}. \tag{24}$$

The velocity of the particle after the next successive collision u_z' is

$$(u_z' - U) = -(u_z'' - U). \tag{25}$$

For an asymmetric vibrator with a sawtooth amplitude function, the steady state requirement $u_z' = u_z$ provides the distribution function equivalent of Eq. (14),

$$f(u_z, z) = n\delta\left[u_z - \left(\frac{3g\langle U\rangle}{\mu}\right)^{1/2} - gz\right]. \tag{26}$$

For a symmetric vibrator, the number conservation condition requires that the flux of particles reflected with a velocity in the interval du_z about u_z is equal to the flux of particles reflected with velocities in the interval u_z' about du_z', where u_z and u_z' are related by Eqs. (24) and (25).

$$N_r(u_z)du_z = N_r(u_z')du_z'. \tag{27}$$

The fluxes of particles reflected with velocities u_z and u_z' are

$$N_r(u_z)du_z = u_z F(u_z)du_z, \tag{28}$$

$$N_r(u_z')du_z' = u_z' F(u_z')\left(1+\frac{4\mu u_z}{3g}\right)du_z. \tag{29}$$

Equating the two fluxes and using a derivation similar to that for the preceding subsection in the weak dissipation limit, the equation for the distribution function is

$$2u_z^* F''(u_z^*) + F'(u_z^*)\left(4+\frac{2u_z^{*3}}{3}\right) + 2u_z^{*2}F(u_z^*) = 0, \tag{30}$$

where $u_z^* = u_z/(\langle U^2\rangle g/\mu)^{1/3}$. The solution for the above equation is not a Maxwell-Boltzmann distribution, but has a slow decay proportional to u_z^{-3} in the limit $u_z \gg 1$.

IV. CONCLUSIONS

The distribution function for a vibrofluidized bed was analyzed in the limit where the dissipation of energy in a collision due to inelasticity or between successive collisions due to viscous drag is small compared to the energy of particles. Two cases, the dense limit where binary collisions are more frequent than particle collisions with the vibrating surface, and the complementary dilute limit where the particle collisions with the vibrating surface are more frequent than binary collisions, were considered. In addition, two types of dissipation mechanisms, inelastic collisions and viscous drag, and two types of amplitude functions for the vibrating surface were considered. It was found that the dependence of the temperature on the velocity of the vibrating surface remains the same for the dense and dilute limits, though the forms of the distribution functions are very different. When the dissipation is due to viscous drag, the scaling of the velocity is sensitive to whether the amplitude function of the vibrating surface is symmetric or asymmetric. In the dilute limit, the form of the distribution function also depends on the dissipation mechanism and the symmetric or asymmetric nature of the vibrating surface.

An examination of the relations in Table I indicates that in the dense limit, the dependence of the temperature on the properties of the granular material is the same in two and three dimensions. The temperature scales as the square of the amplitude of the velocity of the vibrator for a system with inelastic collisions for both symmetric and asymmetric amplitude functions, but the dependence on the coefficient of restitution is different. In particular, the temperature scales as $(1-e^2)^{-1}$ for a symmetric vibrator, but has a much larger value $\propto(1-e^2)^{-2}$ for an asymmetric vibrator. For the case where dissipation is due to viscous drag, the temperature scales as $U_0^{4/3}$ for a symmetric vibrator, and proportional to U_0 for an asymmetric vibrator, where U_0 is the amplitude of the velocity of the vibrator.

The present results provide definite scaling laws for the temperature of a vibrated granular material in the limit where $(mU_0^2/T) \ll 1$, which corresponds to the limit where the dissipation of energy in a collision due to inelasticity, or between successive collisions due to viscous drag, is small compared to the energy of a particle. These results indicate that the scaling of the temperature on the amplitude of the velocity could vary between $T \propto U_0$ and $T \propto U_0^2$ depending on the mechanism of dissipation and the symmetry of the vibrator. These also provide possible explanations for the scaling behavior observed in previous experiments. The scaling of $T \propto U_0^{1.41}$ in the experiments of Warr *et al.* [8] could be because dissipation occurs due to a combination of inelastic collisions and viscous drag, and the exponent is between the values of $T \propto U_0^2$ and $T \propto U_0^{(4/3)}$ for the two cases. However, the same explanation cannot be provided for the simulations of Luding *et al.* [5], and it is possible that the approximation that $T \gg mU_0^2$ is not valid in this case. It would be useful to systematically probe the parameter regime where $(mU_0^2/T) \ll 1$ in simulations to examine whether the predicted scalings for the temperature are observed in this case, and then proceed to the intermediate regime and examine the scaling behavior.

ACKNOWLEDGMENTS

The author would like to thank Dr. S. Warr and Dr. S. Luding for instructive discussions.

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